AN APPROXIMATE ANALYTICAL MODEL FOR APPLICATION TO THE OPERATION OF INTRASHIP TRANSFER OF MATERIAL AT SEA

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THESIS

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ABSTRACT

An approximate analytical queuing model for the intraship transfer of material phase of the replenishment at sea problem is formulated. The physical model has two parallel sets of three service facilities in series that move material to a final stage of three service facilities in parallel. Exponential service times and finite queue space are assumed for each facility, except that the initial facilities always have infinite queues. This implies that the output from an individual facility is approximately Poisson. The mean output rate for the queuing system is obtained. Several parametric studies on critical parameters are performed, with the results presented in graphical format. The model is easily enlarged to allow a general number of facilities in series in any of the parallel paths. Generalization to more than two paths in parallel for the initial stages may be possible.

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I. INTRODUCTION

Underway replenishment (UNREP) of the ships of the United States Navy at sea is a mandatory requirement in maintaining a continuous posture of readiness to meet the present and future global commitments and responsibilities of this country. However, replenishment of ships at sea should detract as little as possible from the primary missions of a fleet. This implies that the goal of the UNREP operation is complete resupply of combatant ships in the minimum possible time.

There are several phases of the replenishment operation,[1] possibly not all distinct. One list of phases might be: Long range planning required to schedule a replenishment both in terms of time and place; the actual movement of ships toward the prescheduled rendezvous; when several supply and combatant ships are involved in a replenishment operation, the scheduling and movement of all the combatant ships through the replenishment cycle; and the actual transfer of material from the hold of the supply ship to designated storerooms on the combatant. It is obvious that all phases require careful attention if the operation is to be efficient. Consequently, all phases are fruitful study areas in a continuing effort to improve UNREP efficiency.

The study reported herein dealt with the actual transfer operation. Perhaps it is the most important phase in that the ships involved are most vulnerable, their capability to maneuver reduced and their

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[1] Throughout the paper the terms UNREP and replenishment operation will imply the entire operation; planning, scheduling, shipmovement, etc., while the term transfer operation denotes the movement of material from hold of the supply ship to storeroom of the combatant.

watertight boundaries open to allow passage of material. Unfortunately in the past it appears that this phase has received less attention than it warrants both from the standpoint of detailed study and the introduction of improved material handling equipment and techniques.

Recent efforts have been made that should considerably improve the overall speed and efficiency of the transfer operation. Improvements include the introduction of new supply ships with the capability to employ the unit load system of material storage and transfer, more and better mechanized material handling equipment on both supply and combatant ships, and a realization of the problem by ship designers which has resulted in improved match of replenishment stations between supply and combatant ships. The unit load concept provides for material to be stored on the supply ship in the configuration for future transfer. Full implementation of the concept will maintain the unit load intact through ultimate storage in the storeroom or magazine of the combatant receiver. Although this concept may never be completely feasible, especially in the case of smaller combatant ships, it should provide a vast improvement to perhaps the most important and demanding of all UNREP operations, the rearming of aircraft carriers.

The transfer operation can be represented as a queuing problem. It is a system of parallel queues in series with movement between the parallel paths. Attempts at reaching an analytic solution are further thwarted by the finite waiting room in the queues involved. Formal efforts to analyze the problem have been few in number. The powerful computer simulation tool has been the general mode of analysis. An early simulation was conducted by Stanford Research Institute [8]. A detailed computer simulation study has also been done by Alfred M. Feiler of Project TRANSIM, University of California at Los Angeles, Department of Engineering. Computer simulation studies are currently being done at Stanford Research Institute and Center for Naval Analyses.

II. BACKGROUND

A. THE TRANSFER OPERATION

Although some transfer of material between ships at sea is presently accomplished by helicopter, the vast majority of material moves between ships via attached horizontal rigs. A typical movement sequence for material transferred at sea would commence with a vertical lift from the hold of the supply ship to the main or transfer deck by means of an elevator or a boom and windlass. Once on the transfer deck the material will be moved to the transfer station by fork lift truck or hand pallet truck. The intraship transfer of material is accomplished by one of several different cable and pulley arrangements. The type employed on a given transfer will depend upon the types of ships involved and the amount and type of material to be transferred. Material landed on the transfer station of the combatant ship is cleared from the area by fork lift or pallet truck. Eventually the transferred material will be transported vertically down to its designated storeroom or magazine by elevator.

The type of movement just discussed employed the unit load concept required by the model developed herein. As noted earlier, many small combatant ships must break up the unit loads into smaller units for movement beyond the transfer station. Many times hand movement is required because of space limitations in the ship's internal passages. The approximate model developed by this study could still be used for this type of movement using a mean service time for the unit load transferred by pieces.

Queues of material and subsequent blockages of material can and do form at several points along this path. On the supply ship, a queue of material will probably exist on the transfer deck near the point where vertical movement from the hold is completed. A queue of unit loads will also exist at the transfer station as material awaits movement to the combatant. The transfer station on the receiving ship is a critical blocking point as transfer of a load from the supply ship to the combatant may not begin until the landing area has been cleared of the previous load. Generally, material cleared from the landing area on the combatant ship is temporarily stored in what is referred to as a marshalling area. Material in the marshalling area is distributed to a strikedown route and then to eventual storage in the assigned storeroom. In many cases, additional queues of material will form along this strikedown route, especially at the top and bottom of vertical movement leg(s).

Much of the material transferred at sea presently is stored as an individual component or packaged with similar items in such a manner that hand movement is possible. These individual components are combined into loads that can be conveniently handled by the transfer rig between the two ships. This load makeup is accomplished either in the hold or on the main deck of the supply ship. The process is essentially reversed on the receiving ship with the breakup of loads commenced early in the storage routine, especially on smaller ships, where it is generally cleared from the landing area piece by piece. Introduction of more and better handling equipment on all ships and movement of material via unit loads through all or most of the cycle will vastly improve the efficiency of the operation. This step forward, although far from universal reality, is in progress.

Careful consideration indicates that the three distinct entities having a major effect on the movement of material through the system are the service or handling rates through all phases, the storage capacity of the various queues that inherently form and the origination and final distribution of the transferred material. The importance of the handling rates is obvious to the extent that discussion here is unnecessary. However, the importance of the queuing (temporary storage) space and the distribution of material requires some explanation. Any amount of free open space on any ship is always scarce, this includes space for temporary storage of transferred material. Perhaps the single exception is the hanger deck on an aircraft carrier, (aircraft carrier transfer stations are located off the hanger deck), however, even here space may be at a premium as it is normally taken up by aircraft in various states of overhaul. Therefore, the transfer process is often slowed because of blockages caused by the inherent space limitations on board ship.

Limited space on ships is also partially responsible for transfer slowdowns that can be attributed to excessive distribution to a single material handling facility, e.g., a vertical strikedown route on the combatant ship. Space available on a ship restricts both the size of storerooms and vertical and horizontal passages leading to the storerooms. In order to provide for convenient and rapid retrieval of a given item, it must be stored in a predesignated place. Further, many items require special storage facilities that are available in only a single location. These special facilities can range from refrigerated storage for fresh and frozen provisions to storage trays and racks for most ordnance. Even if the above difficulties could be overcome, space limitations again would be limiting in that storerooms are generally kept full so that the only storage space available is the designated one. Therefore, it is often impossible to change material destination to accommodate transfer efficiency. A possible solution to blockage on a route would be an alternate route for some of the material. Unfortunately, alternate routes may not always exist.

B. PARALLEL/SERIES QUEUING MODELS

Solutions for a single stage of several parallel queues, with various types of input and service times, have been used for some time. Knowledge of single server queues in series is more limited and is generally restricted to Poisson input to the first stage and exponential service times in all stages with infinite queues allowed. Information in the literature of a combination of queues in series and parallel is virtually nonexistent.

Saaty [6] gives a survey in the area of series queues. The bibliography for this work is nearly complete and he has recently addended the bibliography to include recent contributions in the field [7]. Burke [1] reported the fundamental result for queues in series with Poisson input to the first stage, exponential service time in all stages with infinite queues allowed, showing that the output from each stage and therefore the input to the next stage is also Poisson. Hunt [4] using the assumption of exponential service times did initial work in the area of allowing only finite queues to form before each series facility. Hillier and Boling [2] extended

Hunt's work under the exponential service time assumption and also solved the problem for Erlang service times for each stage.

Hillier and Boling concurrently reported an approximate procedure for determining the mean output rate for a system of exponential service time facilities in series with finite queues allowed before each facility except the initial stage where an infinite queue was assumed to exist. Hillier and Boling compared results obtained with the approximate procedure with the limited exact results obtained. The comparison showed very little error in all cases with the magnitude of the error decreasing with increased allowable queue size.

The Hillier and Boling approximate procedure has the distinct advantage of being computationally feasible, even for very large problems. This approximate procedure formed the basis for the study reported here and it will be discussed and outlined in some detail in the next section.

C. APPROXIMATE PROCEDURE

The important features of the Hillier and Boling [2] procedure, for approximating efficiently the mean output rate (R) for a series of single server exponential service time facilities, may be outlined as discussed below. Assume that there are N facilities in series each with an exponential service rate of μ_i, $i = 1,2, \cdots, N$, and waiting room M_i, $i = 2,3, \cdots, N$, where the M_i's include the item being served and that an infinite queue always exists before the initial facility.

The basis for the procedure comes from the results due to Burke [1] which indicated that when the M_i's are equal to infinity then each service facility would have a Poisson input. Reich [5] showed further that in this case the queue sizes would be independent. This implies that each facility could be analyzed separately using the standard M/M/1 model. The approximate procedure with finite queues allowed is then based on the assumption that the output from each stage is approximately Poisson. Each facility is analyzed individually by using the single-server queuing model M/M/1 with finite queue allowed. Note that R is merely the mean output rate for each of the facilities.

The procedure then assumes that there exists an effective mean arrival rate $\lambda_{eff}^{(j)}$ (when there isn't blocking in the $(j-1)^{st}$ facility) to the j^{th} facility. It is also assumed that the queue lengths before each facility are independent so that the unconditional probability $(1-P_o^{(j-1)})$ that the $(j-1)^{st}$ facility is busy is equal to the conditional probability that it is busy, given that the queue for the j^{th} facility is not full. Thus it follows that,

$$(1) \qquad \lambda_{eff}^{(j)} = \begin{cases} \mu_{j-1}(1-P_o^{(j-1)}) , & \text{if } n \leq M_j \\ 0 , & \text{if } n = M_j+1 \end{cases}$$

where n is the number of customers in the queue for the j^{th} facility (including any customer being held at the $(j-1)^{st}$ facility because of blocking) and $j = 2,3, \cdots ,N$.

The effective service rate for the j^{th} facility, $\mu_{eff}^{(j)}$, will be somewhat less than μ_j, $j=2,3, \cdots ,N-1$, because time spent in holding a customer after service due to blockage in the $(j+1)^{st}$ queue must be included. Now the mean output rate of a single-server queuing system is the product of the mean service rate and the probability that the server is busy. Therefore, it follows that,

$$(2) \qquad \mu_{eff}^{(j)} = R/(1-P_o^{(j)}) \qquad , \text{ for } j = 2,3, \cdots ,N.$$

However, for the final server the customer is always released immediately, i.e., $\mu_{eff}^{(N)} = \mu_N$. Therefore,

(3) $$R = \mu_N (1-P_o^{(N)}).$$

Thus R may be determined if $P_o^{(N)}$ is known.

For a single server queuing system with finite queue, it can be shown that [3]

(4) $$1-P_o^{(j)} = \frac{y_j(1-y_j^{Mj+1})}{1-y_j^{M_j+2}}$$

where for the case of the approximate system,

$$y_j = \lambda_{eff}^{(j)}/\mu_{eff}^{(j)}.$$

It is also clear that since the queue before the first facility is infinite, $P_o^{(1)} = 0$ or $(1-P_o^{(1)}) = 1$.

It is now possible to determine the true value of R by first assuming a trial value of R, call it R*. Since $(1-P_o^{(1)})$ is known, $\lambda_{eff}^{(2)}$ can be determined from equation (1). It then can be used along with R* in equations (2) and (4) to determine $(1-P_o^{(2)})$. Then $1-P_o^{(3)})$, $(1-P_o^{(4)})$, $\cdots$, $(1-P_o^{(N)})$ can be obtained in a similar manner so that R can be calculated from (3). This calculated value of R is compared to R* and a new trial value assumed and the process repeated until the true value of R is obtained when R* - R = 0.

Hillier and Boling have simplified the above procedure for solution by numerical methods and the equations used in this study are presented below:

Combining equations (1) and (2) yields,

$$y_j = \begin{cases} \mu_{j-1}(1-P_o^{(j-1)})(1-P_o^{(j)})/R & , \quad j = 2,3, \cdots , N-1 \\ \mu_{N-1}(1-P_o^{(N-1)})/\mu_N & , \quad j = N \end{cases}$$

then,

$$(5) \qquad 1-P_o^{(j)} = \frac{R}{\mu_{j-1}(1-P_o^{(j-1)})} y_j \ , \ j = 2,3, \cdots, N-1,$$

so that y_j can now be found by combining (4) and (5), as the positive real root of the equation,

$$(6) \qquad y_j \frac{(1-y_j^{M_j+1})}{1-y_j^{M_j+2}} - \frac{R}{\mu_{j-1}(1-P_o^{(j-1)})} y_j = 0.$$

Hillier and Boling show that this root exists and is unique for values of R^* that are sufficiently close to R, the true mean output rate.

Let,

$$f_j(y_j) = \frac{y_j(1-y_j^{M_j+1})}{1 - y_j^{M_j+2}} \ , \ j - 2,3, \cdots, N-1$$

and

$$g_j(R,y_j) = f_j(y_j) - c_j y_j, \ j = 2,3, \cdots, N-1$$

where R is a trial value,

$$c_j = \frac{R}{\mu_{j-1} X^*_{j-1}} \ ,$$

and $X_j^* = f_j(y_j^*)$, y_j^* corresponding to the unique root of $g(R,y_j) = 0$. Finally $_N X_N^*$ yields the calculated value of R where

$$X_N^* = f_N \left(\frac{\mu_{N-1}}{\mu_N} X^*_{N-1}\right).$$

III. FORMULATION OF THE PROBLEM

For the replenishment at sea transfer problem, as with many other physical problems that can be formulated in a queuing theory context, one of the interesting questions that can be asked is, "How long will the operation take?" This question was answered in the formulation for the UNREP transfer problem by determining the mean output rate (R) for the system.

As previously noted, each at sea transfer of material will be somewhat different from a queuing standpoint because of the ships involved, volume and type of material transferred, material origin and destination, types of material handling devices employed, etc. Thus, the number and size of queues formed during any transfer may differ considerably.

A completely general model to represent the transfer at sea operation would require up to five parallel paths from the supply ship to the combatant ship. On each of the parallel paths there are several material handling facilities with associated queues and waiting space on both the sending and receiving ships. Additionally, the general model would require movement of material from one parallel path to another at least once on each ship.

The general model described above may be simplified considerably and still represent accurately many transfer operations. The model developed for this study utilized two parallel paths from the hold of the supply ship to a marshalling area on the combatant ship. From each of the two marshalling areas the material is moved to any one of three strikedown routes. Figure 1 is a pictorial representation of this model. The inclusion of only two parallel paths was done for computational efficiency, noting also that the vast majority of at sea transfers are conducted with but two attached rigs. A generalization of the model for more than two paths is discussed in section VI. Restriction of movement between parallel paths on the supply ship is the other major simplification made for this study. It was felt that such an assumption was not overly restrictive and accurately represents many actual transfers.

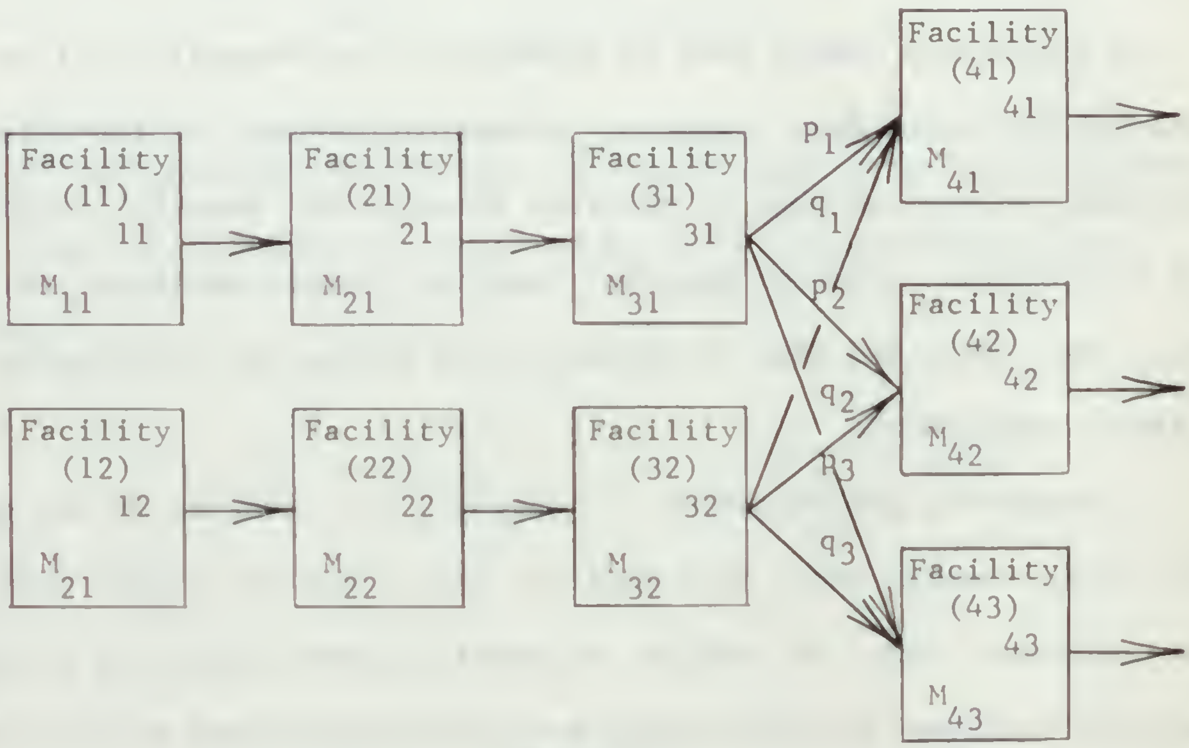


Figure 1

If individual material handling facilities in parallel are referred to as a stage, then the physical location of the first stage queue of the model can represent the hold of the supply ship. The second stage queues are formed by material on the main deck of the supply ship awaiting intraship transfer. The marshalling areas, which will generally be adjacent to the transfer stations on the receiving ship, form the waiting space for the third stage queues. The fourth stage queues may be located physically at any one of several places on the individual strikedown routes to the designated storerooms, e.g., on the transfer deck near an elevator or hatch, or at the bottom of a vertical movement leg.

As stated previously, service of material in the two third stage queues consists of movement to any one of the three strikedown routes. In the model, this movement is governed by two probability distributions p_i and q_i, $i = 1,2,3$, for the upper and lower routes respectively. The distributions are determined as that percent of the material moving across one of the initial routes that is scheduled for eventual storage in a storeroom on the i^{th} strikedown route. Some of the factors that effect this determination are the amount and type of material transferred, the storage of material on the supply ship, the location of the storage space available on the combatant ship, and the availability of alternate routes to final storage. It is felt that the distribution approach as a means of representing the actual movement during transfer is reasonably flexible to cover accurately many transfer operations. A primary consideration in this regard is the fact that material is selected for movement in the most convenient manner, i.e., a load is selected for movement because it is available to the transfer facility or because it can be moved ahead of other unit loads to a nonblocked facility upstream.

Within the model it is assumed that all material is moved as a unit load. Solution for smaller units would overly complicate the problem, however, the unit load assumption is enhanced by the present trend toward actual movement of material in this manner. This assumption had a further and probably a greater impact on obtaining an eventual solution because of the effect it had on the transfer rates.

Since all material moves in this manner it is reasonable to assume that a given facility can handle a unit load of 500 lb. bombs or 1000 lb. bombs or even fresh provisions, for example, at approximately the same rate. Thus the unit load concept helps justify the requirement that each facility must move all types of material at a fixed rate during each transfer investigation. There is of course some allowance for variation, e.g., if two types of material had different service rates for a given stage and each moved on different parallel paths, then the service rates for the two paths could be adjusted to represent more accurately the material being handled.

Notation within the model is such that the first subscript (i) represents the i^{th} stage in series and the second subscript (j) represents the j^{th} parallel element in a stage, e.g., μ_{12} is the service rate for the second parallel facility in the first stage and M_{31} is the number of unit loads that can queue before the first parallel facility in the third stage.

IV. PARALLEL SERIES SOLUTION

The flow of material through the physical model of figure 1 may be represented as in figure 2. If R is the mean output rate, it follows

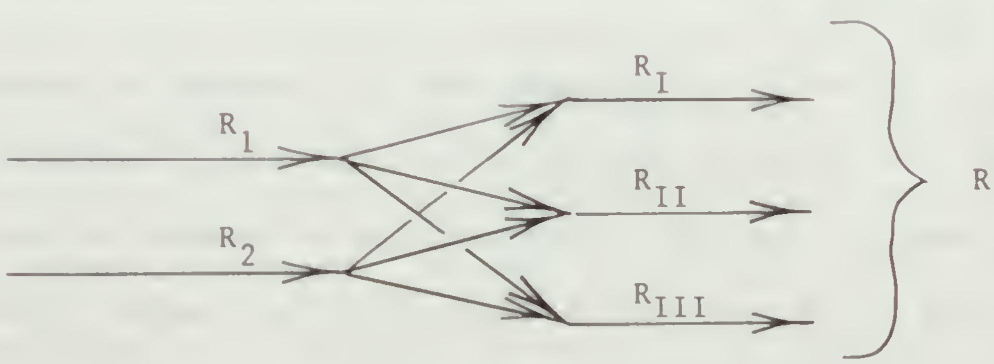


Figure 2

from previous discussion of the p_i and q_i distributions and figure 2 that,

$$R = R_1 + R_2$$

$$R = R_I + R_{II} + R_{III}$$

$$R_I = p_1R_1 + q_2R_2$$

$$R_{II} = p_2R_1 + q_2R_2$$

$$R_{III} = p_3R_1 + q_3R_2$$

where R_1, R_2, R_I, R_{II}, and R_{III} are the mean output rates of their respective routes. For any UNREP that can be represented by this model, the value of the mean output rates will be determined by the various service rates, queue waiting space, and the p_i and q_i distributions.

If the system were symetric, that is if the service rates and queue lengths for the facilities in each stage were the same and the p_i and q_i distributions moved material symetrically to the final stage, then R_1 would equal R_2. A solution to this problem could be easily reached by the Hillier and Boling procedure by considering only half the system and solving for R/2. However, these conditions will not often hold and in general, R_1 will not equal R_2.

When R_1 and R_2 are not equal, the Hillier and Boling approximate procedure cannot be applied directly to the system to obtain a solution for R. The problem is that not only must a trial value for R be assumed, but also values for R_1 and R_2 must be assumed such that $R_1^* + R_2^* = R^*$, where the R^*'s are trial values. Now there are infinitely many combinations of $R = R_1^* + R_2^* = R^*$. The problem thus reduces to finding the values of R_1 and R_2 that will yield the true mean output rate. The true values of the R's will be complicated functions of μ_{ij}, M_{ij}, p_i, and q_i. It will not, however, be necessary to determine this function, as the solution may be reached by simply maximizing the sum $R_1 + R_2$ for a given value of R^*.

Before proceeding to show how the true values of R_1, R_2 and R may be obtained, the Hillier and Boling equations should be adapted to the problem. Assume that for a given problem the true values of R_1, R_2, and R are known. The solution for the output across the upper route may be found by applying the approximate procedure previously outlined. First y^*_{21} may be found as the unique positive real root of

$$g_{21}(R^*_1, y_{21}) = f_{21}(y_{21}) - c_{21}y_{21} = 0$$

where $c_{21} = R^*_1 / \mu_{11}X^*_{11}$, $(X^*_{11} = 1)$. Then,

$$X^*_{21} = f_{21}(y^*_{21}) = \frac{y_{21}(1-y^{*M_{21}+1}_{21})}{1-y^{*M_{21}+1}_{21}}.$$

y^*_{31} and then X^*_{31} may be found in a similar manner. Also, X^*_{32} may be found using the same technique using R^*_2 in place of R^*_1. Then since the fourth stage is the final stage X^*_{4i}, $i = 1,2,3$, can be determined directly as

$$X^*_{4i} = f_{4i}\left(\frac{\mu_{31}X^*_{31}p_i + \mu_{32}X^*_{32}q_i}{\mu_{4i}}\right), \qquad i = 1,2,3$$

so that finally $R = X^*_{41}\mu_{41} + X^*_{42}\mu_{42} + X^*_{43}\mu_{43}$.

The solution procedure for the interesting case of unequal parallel outputs can now be outlined. As before, R^* is assumed and them R^*_1 and R^*_2 are chosen such that $R^*_1 + R^*_2 = R^*$. The problem is then solved as just outlined to obtain a computed value of the mean output rate, R. Using the same value of R^*, R^*_1 and R^*_2 are varied (maintaining the relationship $R_1^* + R_2^* = R^*$) until the maximum computed value of R is found.) As in the original Hillier and Boling procedure, the computed value of R is compared to R^* and if $R = R^*$, the true mean output rate has been found. If R is not equal to R^*, then a new value of R^* is assumed and the procedure repeated until the true mean output rate is obtained. This procedure converges rapidly if two techniques are employed. When selecting a new value for R^*, select $R^*_{(new)} = (R^* + R)/2$. Also when commencing each iteration $R^*_{1(new)}/R^*_{2(new)} = R_1^*/R_2^*$.

Justification for this procedure follows from physical flow laws. It is easy to see that the procedure is correct at the boundaries. For example, assume that the system was such that one of the μ_{ij}'s = 0 in the lower path. Then maximum R will be obtained when $R^* = R_1^*$ and $R_2^* = 0$.

A computer program to solve the problem of figure 1 is presented on page 34. The program was employed to obtain the results presented in section V. The wide range of input data was handled with no difficulty. Average computation time for the IBM 360 was 2 seconds.

V. PRESENTATION OF RESULTS

As noted previously, the three types of parameters within the approximate transfer model that have an effect on the mean output rate are the service rates of the material handling facilities (μ_{ij}'s), the available waiting room in each queue (M_{ij}'s), and the distribution of material between the series facilities as governed by the p_i and q_i distributions. The model can be employed to study UNREP transfer operations in at least two ways. First of all, general parametric studies can be made to determine the effect of varying the parameters in all possible combinations. Also, more specific studies may be made employing known handling rates, queue space and expected material distribution for specific ship types, to determine critical areas that can be improved to increase overall transfer efficiency. A second use for the model would be as an aid to the commander, perhaps even on as low a level as commanding officer of a supply ship, to help him in the planning for replenishment operations. All that would be required is a knowledge of material handling characteristics of the ships involved, the amount of material to be transferred and a small computer which is generally carried on board many supply ships today.

The primary advantages of the approximate procedure over known computer simulation models are the simplicity of the analytical model, the inherent computing efficiency and the ease with which inputs into a computer program may be changed. Possible objections to the model over computer simulation models would include the limitation that different types of material must be transferred through a facility at the same rate and that unit loads of each type of material would occupy the same volume or area of queue space. The exponential service time assumption with its memoryless property, i.e., the probability of a unit completing service in a time interval Δt is constant, no matter how much time has elapsed since the last completion of service, also may be partially objectionable. A final objection may be raised over the final output of the model which is a steady state result, i.e., it is assumed that the system has been in operation for a long time and is in equilibrium. This objection is at least partially overcome by the generally large volume of material transferred during the UNREP's that are of interest and the comparatively small queue sizes allowed, thus assuring that equilibrium conditions will be nearly attained during an operation.

Actual UNREP transfer data was not available in the format required for entry in the model. Therefore, numerical results were necessarily confined to several parametric studies. Graphs of the study results, along with the input data for the constant parameters, is presented in Appendix A.

Parametric studies 1, 2 and 3 all vary the parameter p_1, distribution of material from route R_1 to route R_I, over the range 0.333 to 1.0. The parametric variation in the three studies involves the q_i distribution as follows:

Study one (page 29) - - - $q_i = 1/3, \quad i = 1,2,3$

Study two (page 30) - - - $q_1 = 0.2, \quad q_2 = q_3 = 0.4$

Study three (page 31) - - $q_1 = 0.4, \quad q_2 = q_3 = 0.3$

All three studies indicate the marked decrease in overall transfer rate as congestion at the upper strikedown route (facility (4,1)) increases. As predictable, the decrease is most rapid for study 3 where the flow to facility (4,1) and, therefore, the congestion at the facility is greatest. An additional effect, evident in all three studies, is the difference between R_1 and R_2 that can be attributed to the slight reduction in handling rates of facilities (2,2) and (3,2) and the smaller queue allowed at facility (3,2).

Parametric study 4 gives a good indication of the efficiency that can be gained by increasing the handling rates of the facilities in a single stage. In this case, the service rates of facilities (2,1) and (2,2) were varied together over the range 0.5 to 1.0 loads/min. with all other service rates fixed at 0.5 loads/min. The graph of the study (page 32) indicates that small increases above the system average will have a significant effect. Study 5 varied the queue space in facilities

(3,1) and (3,2) over the range 5 to 10 loads with the remaining queue spaces held fixed at 5 unit loads. Once again a graph of the results (page 33) indicates that small changes above the norm are worthwhile. Comparison of studies 4 and 5 shows that increased service rates have a greater effect on efficiency than increases in allowable queue size. This is understandable and also encouraging, as generally, space on ships will not be available to increase temporary storage to promote increased transfer efficiency.

VI. EXTENSION OF THE MODEL

There are several ways that the specific model developed here can be extended. Unfortunately, the most interesting extension becomes computationally difficult.

The most obvious extension follows directly from the Hillier and Boling [2] series development where the number of stages in series is N. The parallel series model developed here may be similarly extended. Referring to figure 2, this would mean that there could be N_1 and N_2 separate service facilities in the two initial parallel routes and N_I, N_{II}, and N_{III} separate service facilities in the three final routes, where N_1, N_2, N_I, N_{II}, and N_{III} are only restricted to be finite. Solution of this generalization should present no special difficulty.

The interesting extension fron an UNREP point of view would seem to be that which generalizes the number of parallel paths allowed. Such a generalization is represented in figure 3.

In solving such a system, it is first of all evident that if R_1, R_2, $\cdots$, R_n are known, then R_I, R_{II}, $\cdots$, R_M are uniquely determined by the p_{ij} distributions. Where p_{ij} is the distribution of material from the i^{th} initial stage to the j^{th} final stage. Since it must hold that

$$\sum_{j=1}^{P} p_{ij} = 1, \quad i = 1,2, \cdots ,n,$$

and

$$R_I = \sum_{i=1}^{n} p_{i1}R_i$$

$$R_{II} = \sum_{i=1}^{n} p_{i2}R_i$$

$$\vdots \qquad \vdots$$

$$R_M = \sum_{i=1}^{n} p_{iP}R_i.$$

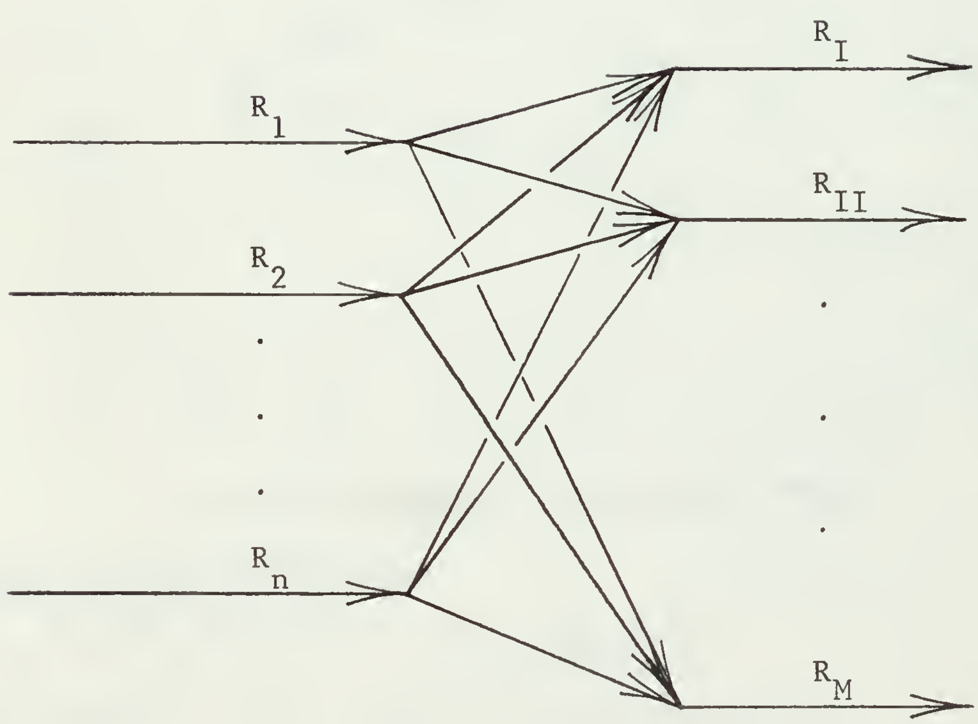
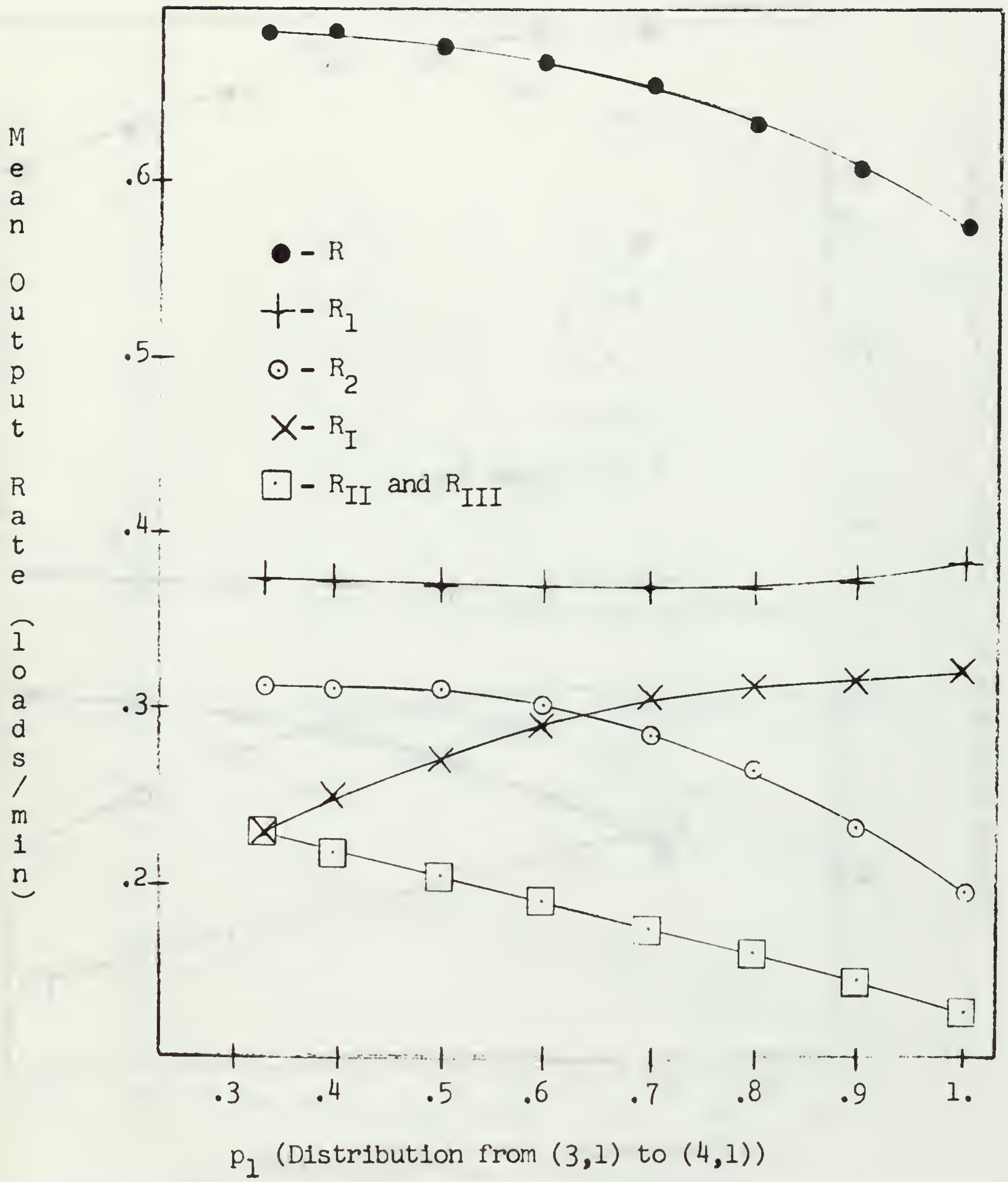


Figure 3

Also, it is not difficult to see that the method of solution presented for n = 2 earlier becomes increasingly complex as n increases. The true computed mean output rate that is obtained by applying the approximate procedure, is found, as before, by determining the combination of $R_1^*+R_2^*+ \cdots +R_n^* = R^*$ that yields maximum R. However, it would seem that an iterative procedure could be developed to solve the problem efficiently for n = 3 or n = 4 on todays high speed computers. Efficient extension beyond this point would probably require knowledge the R_i's explicitly in terms of the service rates, queue lengths and flow distributions.

APPENDIX A



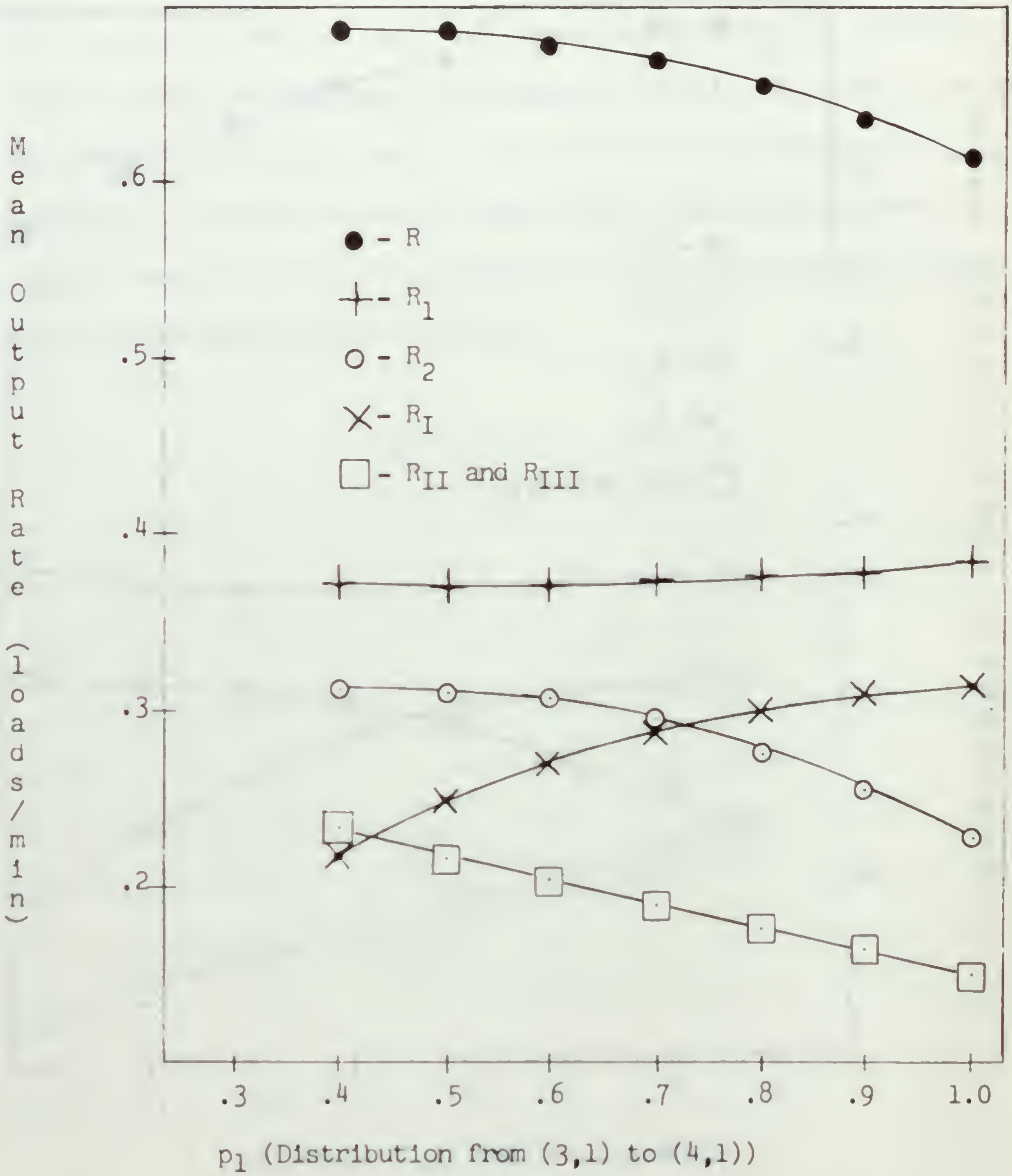
<u>Values of other parameters:</u>

$\mu_{11}, \mu_{12}, \mu_{21}, \mu_{31} = 0.5$ $\mu_{22}, \mu_{32} = 0.4$

$\mu_{41}, \mu_{42}, \mu_{43} = 1/3$ $M_{21}, M_{22}, M_{31}, M_{41}, M_{42}, M_{43} = 5$

$M_{32} = 4$ $q_i = 1/3$, $i = 1,2,3$ $p_2 = p_3 = (1-p_1)/2$

PARAMETRIC STUDY 1.



Values of other parameters:

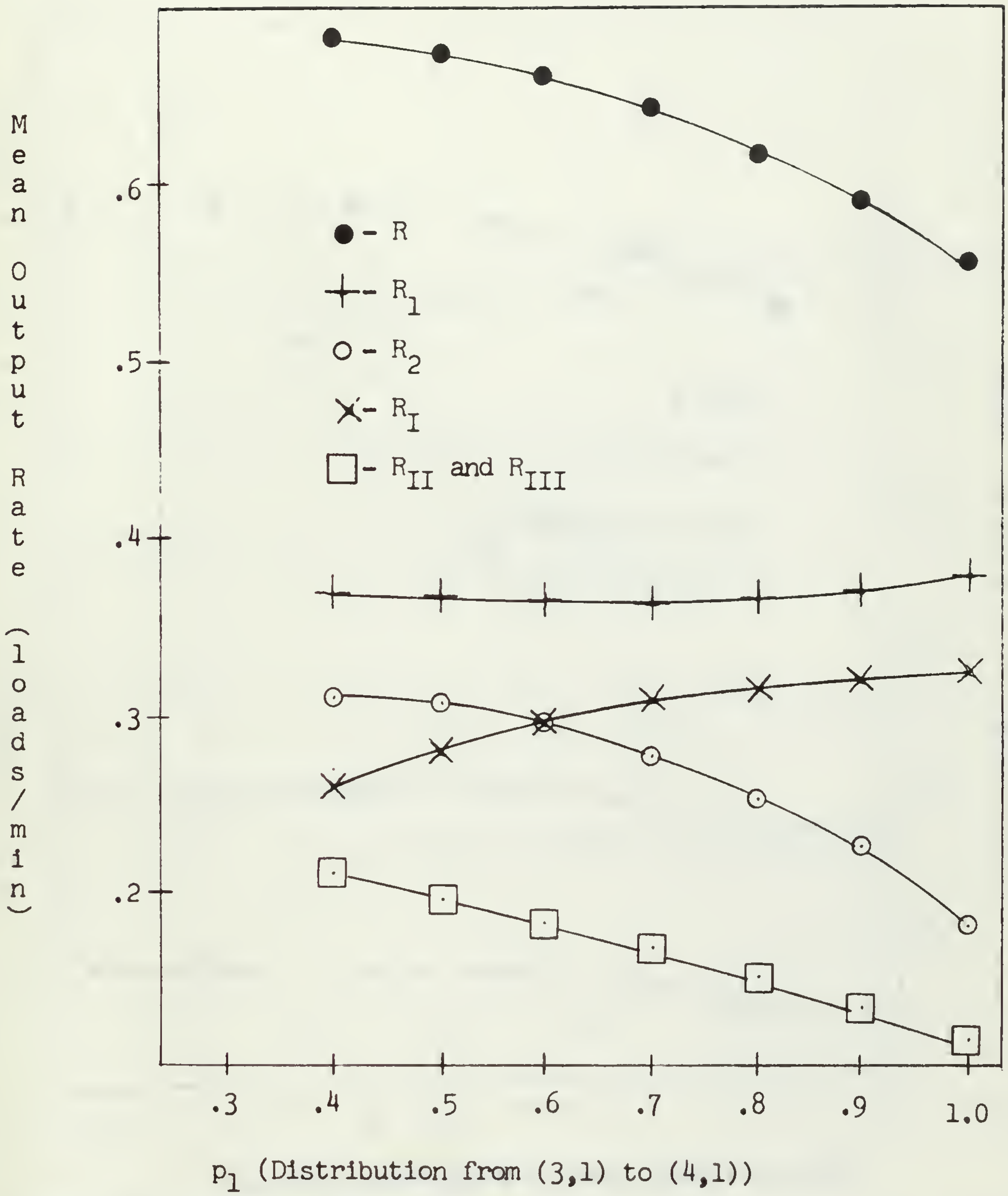
$\mu_{11}, \mu_{12}, \mu_{21}, \mu_{31} = 0.5$ $\mu_{22}, \mu_{32} = 0.4$

$\mu_{41}, \mu_{42}, \mu_{43} = 1/3$ $M_{21}, M_{22}, M_{31}, M_{41}, M_{42}, M_{43} = 5$

$M_{32} = 4$ $p_2 = p_3 = (1-p_1)/2$ $q_1 = 0.2$ $q_2 = q_3 = 0.4$

PARAMETRIC STUDY 2.

APPENDIX A



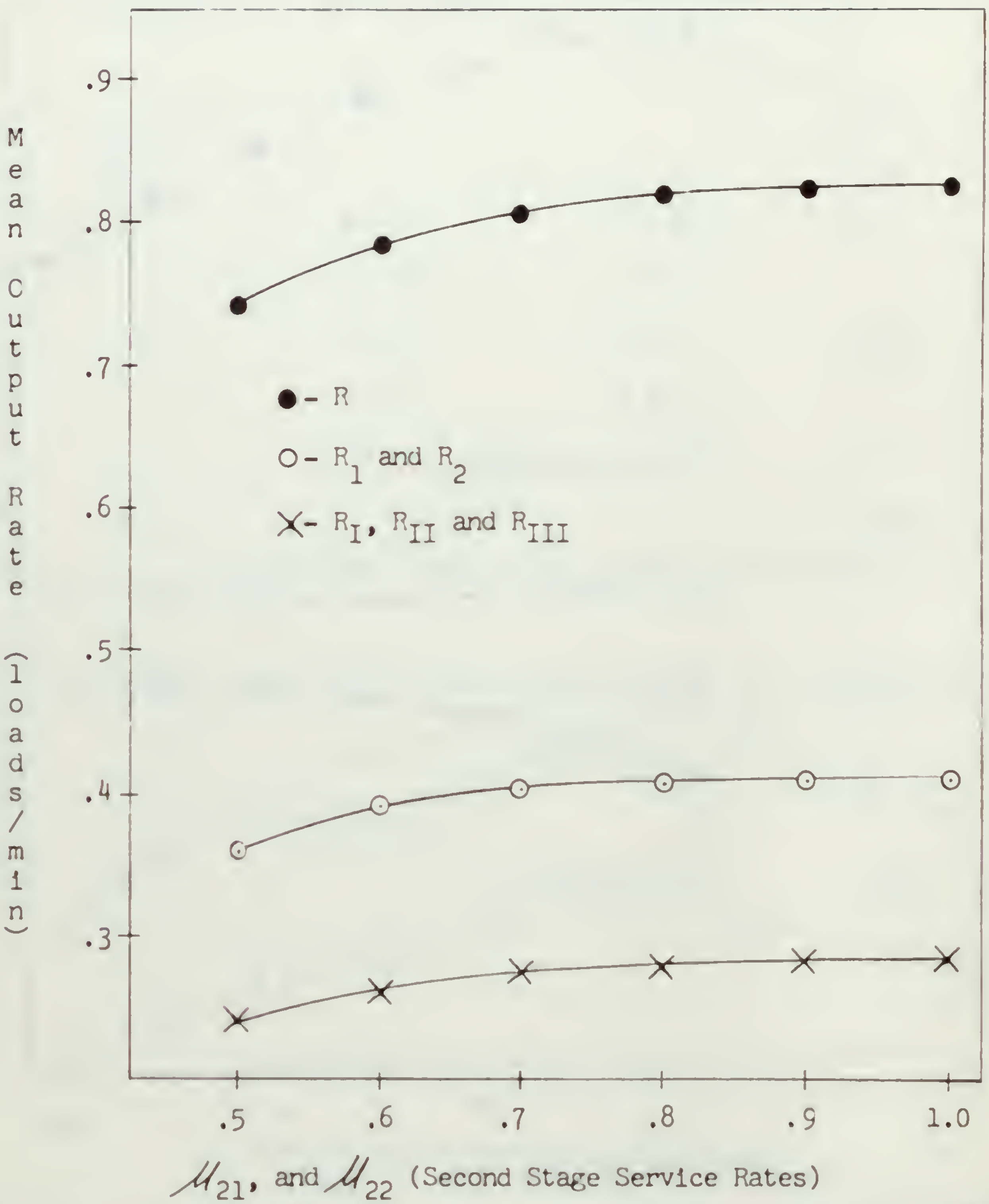
Values of other parameters:

$\mu_{11}, \mu_{12}, \mu_{21}, \mu_{31} = 0.5 \qquad \mu_{22}, \mu_{32} = 0.4$

$\mu_{41}, \mu_{42}, \mu_{43} = 1/3 \qquad M_{21}, M_{22}, M_{31}, M_{41}, M_{42}, M_{43} = 5$

$M_{32} = 4 \qquad p_2 = p_3 = (1-p_1)/2 \qquad q_1 = 0.4 \qquad q_2 = q_3 = 0.3$

PARAMETRIC STUDY 3.



<u>Values of other parameters</u>:

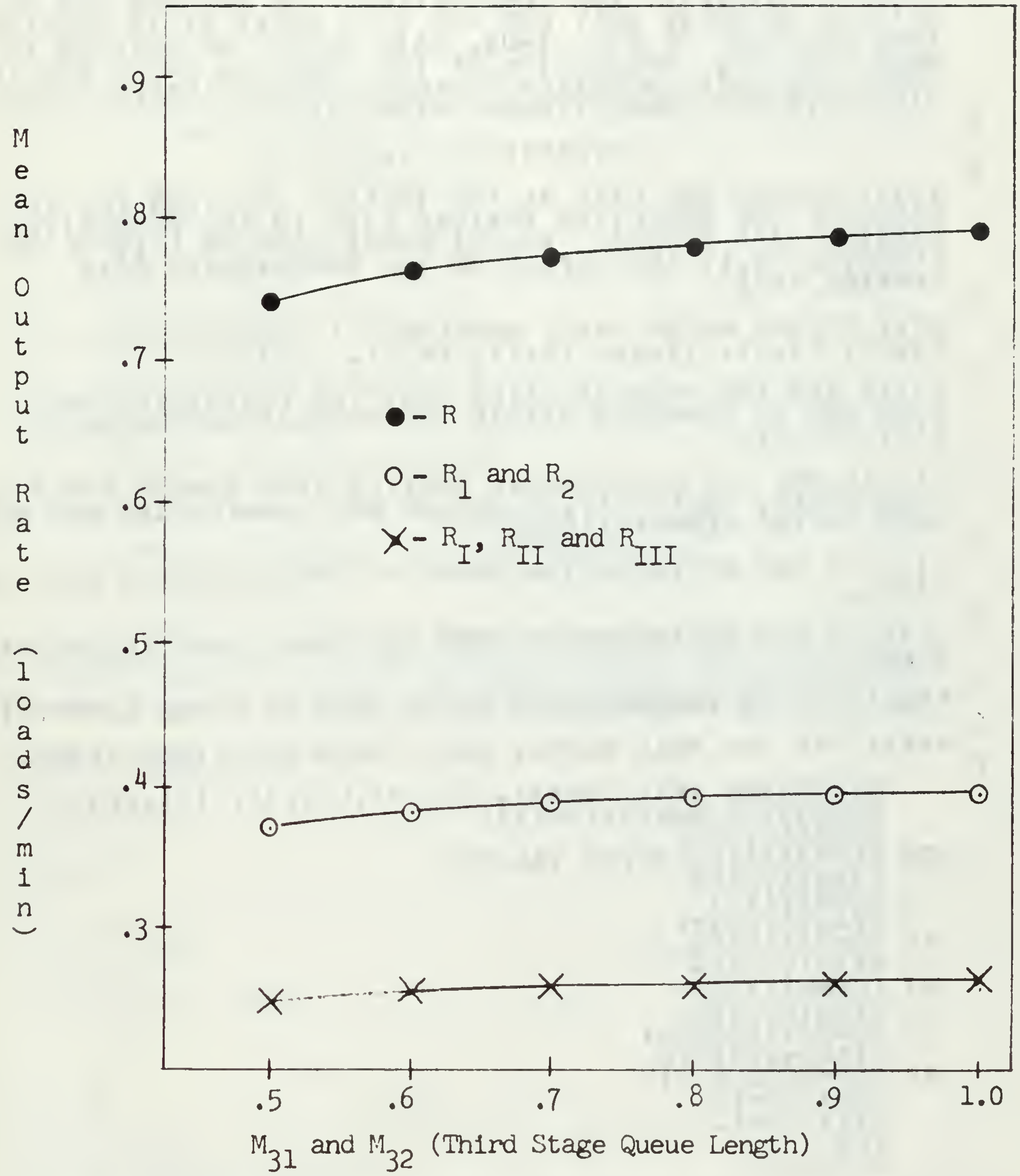
$\mu_{11}, \mu_{12}, \mu_{31}, \mu_{32} = 0.5$ $\qquad \mu_{41}, \mu_{42}, \mu_{43} = 1/3$

$M_{21}, M_{22}, M_{31}, M_{32}, M_{41}, M_{42}, M_{43} = 5$

$p_i = q_i = 1/3, \quad i = 1,2,3$

PARAMETRIC STUDY 4.

APPENDIX A



Values of other parameters:

$\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}, \mu_{31}, \mu_{32} = 0.5$

$\mu_{41}, \mu_{42}, \mu_{43} = 1/3$ $M_{21}, M_{22}, M_{41}, M_{42}, M_{43} = 5$

$p_i = q_i = 1/3, \quad i = 1,2,3$

PARAMETRIC STUDY 5.

```
      COMPUTER PROGRAM TO OBTAIN APPROXIMATE SOLUTION FOR THE
      MEAN OUTPUT RATE FOR THE REPLENISHMENT AT SEA PROBLEM
C
      NOTE: TO SIMPLIFY THE COMPUTATIONS THE PROGRAM HAS BEEN
      WRITTEN TO SOLVE ONLY FOR THE CASE WHERE R1 IS GREATER
      THAN OR EQUAL TO R2.  THUS, DATA SHOULD BE ENTERED TO
      MAKE THIS SO.  INSPECTION OF THE HANDLING RATES FOR REAL
      WORLD DATA WILL NORMALLY INDICATE WHICH PARALLEL PATH
      WILL HAVE THE LARGEST MEAN OUTPUT RATE
C
                        DIMENSIONED VARIABLES
C
      YY(I) VALUES ARE USED AS THE INITIAL SOLUTION FOR THE
      RATIO OF THE EFFECTIVE ARRIVAL RATE TO THE EFFECTIVE
      SERVICE RATE VALUES.  VALUES CORRESPOND TO 1/10 OF THE
      INTERVAL (0,1) AND DEPEND ON THE APPROXIMATE MEAN
      SERVICE RATE.
C
      M(I) DEFINE QUEUE SPACE AVAILABLE AT FACILITIES (2,1)
      (2,2), (3,1), (3,2), (4,1), (4,2), (4,3).
C
      XS(I) ARE THE PROBABILITIES THAT THE FACILITIES ARE
      BUSY AND IS COMPUTED WITHIN THE PROGRAM FOR FACILITIES
      (1,1)-(2,3).
C
      U(I,J) ARE THE EXPONENTIAL SERVICE TIME INPUTS FOR THE
      FACILITIES.  NOTE U(1,3)-U(3,3) ARE DUMMIES AND ARE NOT
      USED IN THE COMPUTATIONS.
C
      P(I) IS THE DISTRIBUTION FROM THE UPPER PATH TO THE FINAL
      STAGE.
C
      Q(I) IS THE DISTRIBUTION FROM THE LOWER PATH TO THE FINAL
      STAGE.
C
      RHO(I) IS AN INTERMEDIATE VALUE USED IN FINAL COMPUTATION.
C
      RR(I) ARE THE MEAN OUTPUT RATES FROM THE FINAL STAGE.
C
          DIMENSION YY(10),M(7),XS(2,3),U(4,3),P(3),Q(3)
          DIMENSION RHO(3),RR(3)
          WRITE(6,600)
      600 FORMAT(T11,'WRITE VALUE')
          READ(5,5)MM
        5 FORMAT(I2)
          READ(4,46)YY
       46 FORMAT(10F6.2)
          READ(4,47)M
       47 FORMAT(7I6)
          READ(4,48)U
       48 FORMAT(12F6.4)
          READ(4,49)P,Q
       49 FORMAT(6F6.4)
          XS(1,1)=1.
          XS(1,2)=1.
          I=1
          DEN=100.
C
      AR IS THE INITIAL TRIAL SOLUTION FOR THE MEAN OUTPUT RATE.
C
          AR=0.6*(U(4,1)+U(4,2)+U(4,3))
C
      R1 IS THE TRIAL VALUE FOR THE UPPER ROUTE
C
          R1=AR/2.
C
      R2 IS THE TRIAL VALUE FOR THE LOWER ROUTE
          R2=AR-R1
C
       63 C=R1/U(1,1)
```

```
      SUBROUTINE SELEC DETERMINES AN INITIAL TRIAL SOLUTION (Y)
      FROM THE TABLE YY FOR THE POLYNOMIAL TO BE SOLVED BY THE
      NEWTON RAPHSON METHOD.
C
      CALL SELEC(C,YY,Y)

      SUBROUTINE APROX SOLVES THE POLYNOMIAL TO DETERMINE THE
      RATIO OF EFFECTIVE ARRIVAL RATE TO EFFECTIVE SERVICE RATE
      (Y) AND THEN COMPUTES XS(I,J).
C
      CALL APROX(C,M(1),Y,XS(2,1))
      C=R2/U(1,2)
      CALL SELEC(C,YY,Y)
      CALL APROX(C,M(2),Y,XS(2,2))
      C=R1/(U(2,1)*XS(2,1))
      CALL SELEC(C,YY,Y)
      CALL APROX(C,M(3),Y,XS(3,1))
      C=R2/(U(2,2)*XS(2,2))
      CALL SELEC(C,YY,Y)
      CALL APROX(C,M(4),Y,XS(3,2))
      A=XS(3,1)*U(3,1)
      B=XS(3,2)*U(3,2)
      DO 60 K=1,3
      RHO(K)=(P(K)*A+Q(K)*B)/U(4,K)
      D=RHO(K)**(M(4+K)+1)
   60 RR(K)=(RHO(K)-D)/(1.-D)*U(4,K)
C
      RS IS THE COMPUTED VALUE OF THE OUTPUT RATE.
C
      RS=RR(1)+RR(2)+RR(3)
C
      THIS SECTION OF THE PROGRAM COMPARES THE TRIAL VALUE AR
      TO THE COMPUTED VALUE RS AND ITERATES UNTIL THE DIFFERENCE
      IS LESS THAN .001.  THE SECTION ALSO CAUSES ITERATIONS TO
      BE MADE TO FIND THE VALUES OF R1 AND R2 THAT YIELD MAX RS.
C
      IF(I.EQ.1)GO TO 61
      IF(RS.LT.RS1)GO TO 62
      RS1=RS
   64 R1=R1+1./DEN
      R2=AR-R1
      GO TO 63
   61 RS1=RS
      I=2
      GO TO 64
   62 R1=R1-1./DEN
      R2=AR-R1
      IF(I.NE.2)GO TO 65
      DEN=1000.
      I=3
      GO TO 64
   65 I=1
      DEN=100.
      S=R1/R2
      T=ABS(RS-AR)
      IF(T.LE.0.002)GO TO 66
      IF(RS.GT.AR)GO TO 67
      AR=AR-T/2.
   69 IF(R1.EQ.R2)GO TO 68
      R3=ABS(R1-R2)
      IF(R3-.1)68,68,70
   70 R1=AR/2.+R3-.1
      R2=AR-R1
      GO TO 63
   67 AR=AR+T/2.
      GO TO 69
   68 R1=AR/2.
```

```
      R2=AR-R1
      GO TO 63
   66 WRITE(MM,55)RS,R1,R2,RR
   55 FORMAT(//T5,'QUEUE OUTPUT IS',T23,F8.5//T5,'PARALLEL
      OUTPUTS ARE',T28,2F8.5//T5,'TERMINAL OUTPUTS ARE',
      T28,3F8.5)
      WRITE(MM,52)U
   52 FORMAT(//T44,'HANDLING RATES'//(1H0,T10,12F7.4))
      WRITE(MM,53)M
   53 FORMAT(//T40,'MAXIMUM QUEUE LENGTHS'//(1H0,T25,7I7))
      WRITE(MM,54)P,Q
   54 FORMAT(//T37,'DISTRIBUTION TO FINAL STAGE'//
      (1H0,T25,6F8.4))
      STOP
      END


      SUBROUTINE APROX(VAL,N,Y,X)
      AN=N
   30 A=Y**N
      B=A*Y
      C=(Y-B)/(1.-B)-VAL*Y
      D=(1.-(AN+1.)*A+AN*B)/(1.-B)**2-VAL
      Y1=Y-(C/D)
      IF(ABS(Y1-Y)-.0001)20,20,22
   22 Y=Y1
      GO TO 30
   20 X=(Y-B)/(1.-B)
      RETURN
      END


      SUBROUTINE SELEC(C,A,B)
      DIMENSION A(10)
      DO 25 I=1,10
      IF(C-FLOAT(I)/10.)23,23,25
   25 CONTINUE
   23 B=A(I)
      RETURN
      END
C
C
                               SAMPLE DATA DECK
C
  YY(I)  INITIAL TRIAL SOLUTION FOR NEWTON RAPHSON POLY-
  NOMIAL SOLUTION,  IT DEPENDS ON THE VALUE OF AR ASSUMED.
YY(1) YY(2) YY(3) YY(4) YY(5) YY(6) YY(7) YY(8) YY(9) YY(10)
 20.00  6.00  4.00  2.80  2.20  1.80  1.50  1.20  0.90  0.60
C
  M(I)  MAX QUEUE LENGTHS
    M21   M22   M31   M32   M41   M42   M43
     5     5     5     4     5     5     5
C
  U(I,J) - HANDLING RATES (NOTE FOR ACTUAL INPUT ALL DATA IS
  ON A SINGLE CARD WITH 12F6.3 FORMAT).
  U11   U21   U31   U41   U12   U22   U31   U42   U13   U23
 .5000 .5000 .5000 .3333 .5000 .4000 .4000 .3333 .5000 .5000
  U33   U43
 .5333 .3333
C
  P(I) AND Q(I) DISTRIBUTIONS
  P(1)  P(2)  P(3)  Q(1)  Q(2)  Q(3)
 .3333 .3333 .3333 .3333 .3333 .3333
```

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ABSTRACT

An approximate analytical queuing model for the intraship transfer of material phase of the replenishment at sea problem is formulated. The physical model has two parallel sets of three service facilities in series that move material to a final stage of three service facilities in parallel. Exponential service times and finite queue space are assumed for each facility, except that the initial facilities always have infinite queues. This implies that the output from an individual facility is approximately Poisson. The mean output rate for the queuing system is obtained. Several parametric studies on critical parameters are performed, with the results presented in graphical format. The model is easily enlarged to allow a general number of facilities in series in any of the parallel paths. Generalization to more than two paths in parallel for the initial stages may be possible.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Replenishment at Sea						
Underway Replenishment						
UNREP						
Series/Parallel Queues						

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